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A note on effects of rational bubble on portfolios

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HIGHLIGHTS

- This article introduces one bubble asset and one bubble free asset.
- We argues that inferior goods can occur for bubble free asset.
- Giffen behavior can also appear for bubble free asset.

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ABSTRACT

In general, demand increases in wealth and decreases in price in microeconomics. We thereby propose a completely different perspective. By establishing expected utility function of investors, this article introduces one rational bubble asset and one bubble free asset in portfolios and focuses on the effects of bubble on investment portfolios from wealth and price perspectives. All conclusions are obtained by theoretical analysis with microeconomics theory. We argue that inferior goods and Giffen behavior can occur for the bubble free asset in microeconomic fields. The results can help investors to recognize bubble assets and bubble free assets more scientifically. Both bubble and bubble free assets can be inferior goods under some conditions, so we cannot to say which asset better than the other one absolutely.

1. Introduction

Bubble has crucial effects on economy, such as stimulating economic growth [1,2], eliminating inefficient investment [3], yielding economic crisis [4], monetary policy [5] and so on. Compared with other assets, bubbles start randomly and without cost and do not produce any outputs. In practice, many other factors yield bubble assets in reality. For example, Santos and Woodford [6] pointed out that rational asset pricing bubbles may arise in an intertemporal competitive equilibrium framework. Hugonnier [16] recently argued that portfolio constraints give rising to rational asset pricing bubbles.

In microeconomics, Samuelson [7] initially noted that bubble is part of competitive economy. Tirole [3] further proved that bubbles can eliminate the inefficient investment and established the overlapping generation framework about bubbles. Doi et al. [15] identified Giffen behavior without relation with wealth. Chen and He [8] analyzed traders' heterogeneity and its impacts on the formation of bubble in financial markets with a multi-agent perspective. Interestingly, Kubler et al.

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[9] recently argued that the risk free asset may lead to Giffen behavior and acts as inferior goods. This paper follows the interesting framework of Kubler et al. [9] to examine whether bubble free asset behaves Giffen.

In macroeconomics, rational bubbles have major effects on the international trades, economic growth, business cycles and so on. Ventura [10] recently followed Tirole's framework and further explored the theory of rational bubbles in international trades. In general, bubbles absorb liberating resources and improve allocation of investment. Martin and Ventura [10] showed that rational bubble stimulates economic growth. Recently, Luik and Wesselbaum [11] argued that a bursting bubble creates large and long-lasting real effects in macroeconomics.

The existing literature neglects the effects of bubbles on investment portfolios in microeconomics and this paper proposes to fills in this gap to address the effects of bubbles on portfolios and shows that rational bubbles can also yield inferior goods and Giffen behavior for bubble free asset.

The rest of this paper is organized as follows: Section 2 establishes the model with both the bubble asset and bubble free asset. Section 3 addresses the model. We deduce that bubble free asset may cause Giffen behavior and act as inferior good. Some remarks are presented in the final section.

2. Model

Here we establish the model with two assets. One is a bubble asset and the other is bubble free. Bubble is defined as the difference between the market price and the market fundamental [3]. The payoff of the unit bubble free asset is $\xi_0 > 0$ and the payoff of the unit bubble asset is ξ, where $\xi = b$ is a random variable. In a pyramid scheme, without loss of generality, we assume the initial bubbles to be $b_0 = 0$. The bubble is assumed to be a stochastic variable $b \in [0, \bar{b}]$ with the density function $f(b) \geq 0$, where $\int_0^{\bar{b}} f(b)db = 1$. The corresponding distribution function is $F(b) = \int_0^b f(\zeta)d\zeta$. The price of the bubble asset is p_b and the price of the bubble free asset is p_f. Denote $\omega > 0$ to be the initial wealth. Note $n \geq 0$ and n_f to be the number of units of the bubble asset and bubble free asset, respectively. The contingent claims are $c = \xi_0 n_f + \xi n$. The utility function with contingent claims c is $u(c)$, which is continuously differentiable of second-order. The expected utility function is

$$E[u(\xi)] = \int_0^{\bar{b}} u(c(\xi))f(\xi)d\xi. \tag{1}$$

The asset demand is subject to

$$pn + p_f n_f = \omega. \tag{2}$$

The investor maximizes function (1) by n and n_f. Moreover, we always assume that $u'(\cdot) > 0$ and $u''(\cdot) < 0$, or all consumers are all risk aversion. The equilibrium depends on the properties of the utility function. We also note that the demand of bubble free asset n_f may be positive, negative or zero, while the demand of bubble asset is non-negative. Furthermore, we assume $\bar{b} > \xi_0$.

3. Primary results

Here the model of portfolios is analyzed. By Eq. (2) and the assumption $u'(\cdot) > 0$ and $u''(\cdot) < 0$, there exists the unique solution to (1)–(2) and we have

$$\frac{\partial E[u(\xi)]}{\partial n} - \lambda p = \frac{\partial[\int_0^{\bar{b}} u(\xi_0 n_f + \xi n)f(\xi)d\xi]}{\partial n} - \lambda p = 0, \tag{3}$$

$$\frac{\partial E[u(\xi)]}{\partial n_f} - \lambda p_f = \frac{\partial[\int_0^{\bar{b}} u(\xi_0 n_f + \xi n)f(\xi)d\xi]}{\partial n_f} - \lambda p_f = 0. \tag{4}$$

In Eqs. (3) and (4), $\lambda \geq 0$ is the Lagrangian multiplier. Eqs. (2), (3) and (4) are restated as

$$g_1(\omega, p, n, p_f, n_f) = \omega - (pn + p_f n_f) = 0, \tag{5}$$

$$\begin{aligned} g_2(\omega, p, n, p_f, n_f) &= p\frac{\partial[\int_0^{\bar{b}} u(\xi_0 n_f + \xi n)f(\xi)d\xi]}{\partial n_f} - p_f\frac{\partial[\int_0^{\bar{b}} u(\xi_0 n_f + \xi n)f(\xi)d\xi]}{\partial n} \\ &= p\xi_0 \int_0^{\bar{b}} u'(\xi_0 n_f + \xi n)f(\xi)d\xi - p_f \int_0^{\bar{b}} \xi u'(\xi_0 n_f + \xi n)f(\xi)d\xi = 0. \end{aligned} \tag{6}$$

The second equality comes from the hypothesis of the continuously differentiable of second-order for $u(c)$. Denote the equilibrium to be (n^*, n_f^*), implicit function theorem, Eqs. (5) and (6) jointly imply

$$n_f^* = \frac{\det\begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ -\omega & -g_2(\omega, p, 0, p_f, 0) \end{bmatrix}}{\det\begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix}} = \frac{\frac{\partial g_2}{\partial n}\omega - \frac{\partial g_1}{\partial n} g_2(\omega, p, 0, p_f, 0)}{\det\begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix}}, \tag{7}$$

$$n^* = \frac{\det\begin{bmatrix} -\omega & -g_2(\omega, p, 0, p_f, 0) \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix}}{\det\begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix}} = \frac{-\frac{\partial g_2}{\partial n_f}\omega + \frac{\partial g_1}{\partial n_f} g_2(\omega, p, 0, p_f, 0)}{\det\begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix}}. \tag{8}$$

Actually, the equilibrium solution (n^*, n_f^*) is determined by many factors, such as the distributed function, utility function and so on.

3.1. Wealth effects

Here we discuss the effects of wealth on the demand of the bubble asset. In general, a normal good's derivative with respect to wealth is positive. Given the wealth and prices, the bubble free asset is inferior if $\frac{\partial n_f^*}{\partial \omega} < 0$.

By Eqs. (5) and (6), we have the following conclusions

Proposition 1. *We have* $\frac{\partial n_f^*}{\partial \omega}\begin{cases} < 0 & \int_0^{\bar{b}} (p\xi_0\xi - p_f\xi^2)u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi < 0 \\ = 0 & \int_0^{\bar{b}} (p\xi_0\xi - p_f\xi^2)u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi = 0 \\ > 0 & \int_0^{\bar{b}} (p\xi_0\xi - p_f\xi^2)u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi > 0. \end{cases}$

Therefore, the bubble free asset is inferior good if and only if $p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi < p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi$.

Proof. See in Appendix. ■

Remarks. Proposition 1 analyzes the characteristics of bubble free asset from wealth perspectives. $p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi < p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi$ implies inferior good for bubble free asset. This conclusion seems interesting. When p is much larger than p_f, $\int_0^{\bar{b}} (p\xi_0\xi - p_f\xi^2)u''(\xi_0 n_f^* + \xi n^*)f(\xi)d\xi < 0$ holds. That means the demand of bubble free asset will decrease with wealth when the price of bubble assets is much higher than that of bubble free assets. The above proposition presents the condition that the bubble free asset is inferior and shows that bubble free asset is not necessarily normal goods. Here a criterion for the authority to judge the bubble is supported.

Moreover, the wealth effect to the demand of bubble asset is

$$\begin{aligned} \frac{\partial n^*}{\partial \omega} &= -\frac{\det\begin{bmatrix} \frac{\partial g_1}{\partial \omega} & \frac{\partial g_2}{\partial \omega} \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix}}{\det J} = -\frac{\det\begin{bmatrix} 1 & 0 \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix}}{\det J} \\ &= \frac{p_f \int_0^{\bar{b}} \xi_0\xi u''(\xi_0 n_f + \xi n)f(\xi)d\xi - p\xi_0^2 \int_0^{\bar{b}} u''(\xi_0 n_f + \xi n)f(\xi)d\xi}{\det J} \\ &= \frac{\xi_0 \int_0^{\bar{b}} (p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n)f(\xi)d\xi}{\det J}. \end{aligned} \tag{9}$$

By the similar way to Proposition 1, formula (9) indicates the following conclusions.

Proposition 2. $\frac{\partial n^*}{\partial \omega}\begin{cases} >0 & \int_0^{\bar{b}}(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n)f(\xi)d\xi > 0 \\ =0 & \int_0^{\bar{b}}(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n)f(\xi)d\xi = 0 \\ <0 & \int_0^{\bar{b}}(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n)f(\xi)d\xi < 0. \end{cases}$

Furthermore, the relationship $p\frac{\partial n^*}{\partial \omega} + p_f\frac{\partial n_f^*}{\partial \omega} = 1$ holds.

Proof. See in Appendix. ■

Remarks. In general, bubble assets are risk goods and have important effects on economic development. Proposition 2 analyzes the characteristics of bubble asset from wealth perspectives. The demand for bubble assets increases with wealth under the condition that $\int_0^{\bar{b}}(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n)f(\xi)d\xi > 0$, which means bubble assets can be normal goods under certain condition. Therefore, we should treat it rationally. Moreover, the sum of the wealth effects with the weighted price is exact one. This can also be induced by Eq. (2). Interestingly, wealth effects of the one goods inhabit those of the other goods.

In summary, from wealth perspective, the above two propositions express a viewpoint that bubble assets are not necessarily inferior goods and bubble free assets may not be normal goods, too. Under some conditions, bubble free assets are inferior goods and bubble assets are normal goods. The results can help investors to make rational asset portfolios.

3.2. Price effects

Here we address the effects of price on the demand of the bubble free asset. According to implicit function theorem, Eqs. (5) and (6) indicate

Proposition 3. $\frac{\partial n_f^*}{\partial p_f}\begin{cases} >0 & \int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n) - pu'(\xi_0 n_f + \xi n)]f(\xi)\xi d\xi > 0 \\ =0 & \int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n) - pu'(\xi_0 n_f + \xi n)]f(\xi)\xi d\xi = 0 \\ <0 & \int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n) - pu'(\xi_0 n_f + \xi n)]f(\xi)\xi d\xi < 0. \end{cases}$

The bubble free asset is inferior good under the following condition

$$\int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f + \xi n) - pu'(\xi_0 n_f + \xi n)]f(\xi)\xi d\xi > 0.$$

Proof. See in Appendix. ■

Remarks. The above conclusions describe the effects of the price on the demand of bubble free asset. Interestingly, bubble free asset may show Giffen behavior. Meanwhile, the condition to yield the bubble free asset being Giffen good is given.

We further address the price effect of the demand of the bubble asset. Eqs. (5) and (6) manifest

$$\begin{aligned}\frac{\partial n^*}{\partial p} &= -\frac{\det\begin{bmatrix}\frac{\partial g_1}{\partial p} & \frac{\partial g_2}{\partial p}\\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f}\end{bmatrix}}{\det J} = -\frac{\det\begin{bmatrix} -n & \xi_0\int_0^{\bar{b}} u'(\xi_0 n_f + \xi n)f(\xi)d\xi \\ -p_f & \begin{matrix} p\xi_0^2\int_0^{\bar{b}} u''(\xi_0 n_f + \xi n)f(\xi)d\xi \\ -p_f\int_0^{\bar{b}} \xi_0\xi u''(\xi_0 n_f + \xi n)f(\xi)d\xi\end{matrix}\end{bmatrix}}{\det J} \\ &= \frac{pn^*\xi_0^2\int_0^{\bar{b}} u''(\xi_0 n_f + \xi n)f(\xi)d\xi - p_f n^*\int_0^{\bar{b}}\xi_0\xi u''(\xi_0 n_f + \xi n)f(\xi)d\xi - p_f\xi_0\int_0^{\bar{b}} u'(\xi_0 n_f + \xi n)f(\xi)d\xi}{\det J} \\ &= \frac{\xi_0\int_0^{\bar{b}}[n^*(p\xi_0 - p_f\xi)u''(\xi_0 n_f + \xi n) - p_f u'(\xi_0 n_f + \xi n)]f(\xi)d\xi}{\det J}.\end{aligned} \tag{10}$$

Eq. (10) indicates the following results, in which the proof is omitted.

Proposition 4. *We have* $\frac{\partial n^*}{\partial p}\begin{cases} >0 & \int_0^{\bar{b}}[n^*(p\xi_0-p_f\xi)u''(\xi_0 n_f+\xi n)-p_f u'(\xi_0 n_f+\xi n)]f(\xi)d\xi>0 \\ =0 & \int_0^{\bar{b}}[n^*(p\xi_0-p_f\xi)u''(\xi_0 n_f+\xi n)-p_f u'(\xi_0 n_f+\xi n)]f(\xi)d\xi=0 \\ <0 & \int_0^{\bar{b}}[n^*(p\xi_0-p_f\xi)u''(\xi_0 n_f+\xi n)-p_f u'(\xi_0 n_f+\xi n)]f(\xi)d\xi<0. \end{cases}$

The bubble assets are normal goods if and only if

$$\int_0^{\bar{b}}[n^*(p\xi_0-p_f\xi)u''(\xi_0 n_f+\xi n)-p_f u'(\xi_0 n_f+\xi n)]f(\xi)d\xi<0.$$

Remarks. Proposition 4 shows the demand of the bubble assets will decrease with price under a certain condition but arise with price under some other condition. That means rational bubble assets may occur when the price falls. Meanwhile, the condition is given by Proposition 4.

In summary, we find the interesting phenomena that inferior goods can occur for bubble free asset, while bubble asset can also be treated as normal goods.

3.3. Example

Here an example is listed to illustrate the above theoretic conclusions. Considering HARA (hyperbolic absolute risk aversion) utility function $u(x)=-(x-a)^{-\delta}/\delta$, where $\delta>-1$ and $a>0$, we have

Example 1. For HARA utility function $u(x)=-(x-a)^{-\delta}/\delta$, we have $u'(x)=(x-a)^{-\delta-1}$ and $u''(x)=-(\delta+1)(x-a)^{-\delta-2}$. Let $a=0$, $\xi_0=1$, $\bar{b}=2$ and $p_f=1$. In this case, we have

$$g_1(\omega,p,n,p_f,n_f)=\omega-(pn+n_f)=0,$$

$$g_2(\omega,p,n,p_f,n_f)=p\int_0^2(n_f+\xi n)^{-\delta-1}f(\xi)d\xi-\int_0^2\xi(n_f+\xi n)^{-\delta-1}f(\xi)d\xi=0.$$

We further achieve $\frac{\partial n_f^*}{\partial \omega}\begin{cases} <0 & \int_0^2(p\xi-\xi^2)(\delta+1)(n_f^*+\xi n^*)^{-\delta-2}f(\xi)d\xi>0 \\ =0 & \int_0^2(p\xi-\xi^2)(\delta+1)(n_f^*+\xi n^*)^{-\delta-2}f(\xi)d\xi=0 \\ >0 & \int_0^2(p\xi-\xi^2)(\delta+1)(n_f^*+\xi n^*)^{-\delta-2}f(\xi)d\xi<0 \end{cases}$ and

$$\frac{\partial n_f^*}{\partial p_f}\begin{cases} >0 & \int_0^2[-n_f^*(\xi-p)(\delta+1)(n_f^*+\xi n^*)^{-\delta-2}-p(n_f+\xi n)^{-\delta-1}]f(\xi)\xi d\xi>0 \\ =0 & \int_0^2[-n_f^*(\xi-p)(\delta+1)(n_f^*+\xi n^*)^{-\delta-2}-p(n_f+\xi n)^{-\delta-1}]f(\xi)\xi d\xi=0 \\ <0 & \int_0^2[-n_f^*(\xi-p)(\delta+1)(n_f^*+\xi n^*)^{-\delta-2}-p(n_f+\xi n)^{-\delta-1}]f(\xi)\xi d\xi<0. \end{cases}$$

When p is large enough, the demand of the bubble free asset is positive. Furthermore, the bubble free asset is inferior and performs Giffen behavior. This example supports the above theoretic conclusions.

Moreover, some real world example also supports the above conclusions. For example, before the collapse of the United States housing bubbles in 2006 and 2007, many bubble-free assets are treated as inferior goods and perform Giffen behavior (Schwartz, 2009; Phillips and Yu, 2011).

In 2006 and early 2007 of China, Chinese stock market is a bull market. The price of stock soared, and many sock investors even sold their houses to invest in stock. The stock bubble yields many bubble free assets to be inferior and with Giffen goods (Jiang et al., 2010).

4. Concluding remarks

This paper highlights bubble asset and, the wealth effects and price effects on the demand of both the bubble asset and the bubble free asset are all captured. Interesting, we argue that the bubble free asset may act as inferior goods and performs Giffen behavior. The result can help investors to recognize bubble assets and bubble free assets more scientifically. These theoretic conclusions are consistent with those in Kubler et al. [9]. Compared with Kubler et al. [9], this paper highlights bubble asset while Kubler et al. [9] focuses on risk asset. Moreover, this paper induces inferior goods and Giffen behavior under general situation. Certainly, some limitations also exist. For example, this paper does not consider the problem of asset pricing. Besides, empirical support may help a lot to our theoretical conclusions if empirical data can be collected. How to choose best asset portfolios between bubble free assets and bubble assets is also an important issue. These topics may be further studied in the future.

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Appendix

Proof of Proposition 1. We first show that functions (5)–(6) are concave. The Jacobian matrix to functions (5)–(6) is listed as follows $J = \begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ \frac{\partial g_1}{\partial n_f} & \frac{\partial g_2}{\partial n_f} \end{bmatrix} = \begin{bmatrix} -p & \begin{array}{c} p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f + \xi n) f(\xi) d\xi \\ -p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f + \xi n) f(\xi) d\xi \end{array} \\ -p_f & \begin{array}{c} p\xi_0^2 \int_0^{\bar{b}} u''(\xi_0 n_f + \xi n) f(\xi) d\xi \\ -p_f \int_0^{\bar{b}} \xi_0 \xi u''(\xi_0 n_f + \xi n) f(\xi) d\xi \end{array} \end{bmatrix}$.

The above formulation and $u''(\cdot) < 0$ imply the relationship

$$\det J = -p^2\xi_0^2 \int_0^{\bar{b}} u''(\xi_0 n_f + \xi n) f(\xi) d\xi + 2pp_f \int_0^{\bar{b}} \xi_0 \xi u''(\xi_0 n_f + \xi n) f(\xi) d\xi - p_f^2 \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f + \xi n) f(\xi) d\xi$$
$$= -\int_0^{\bar{b}} u''(\xi_0 n_f + \xi n) f(\xi)(p\xi_0 - p_f \xi)^2 d\xi > 0.$$

By Functions (5)–(6), implicit function theorem indicates the following formulation

$$\frac{\partial n_f^*}{\partial \omega} = -\frac{\det \begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ \frac{\partial g_1}{\partial \omega} & \frac{\partial g_2}{\partial \omega} \end{bmatrix}}{\det J} = -\frac{\det \begin{bmatrix} \frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n} \\ 1 & 0 \end{bmatrix}}{\det J} \tag{A.1}$$
$$= \frac{p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f + \xi n) f(\xi) d\xi - p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f + \xi n) f(\xi) d\xi}{\det J}.$$

Therefore, $\frac{\partial n_f^*}{\partial \omega} \begin{cases} < 0 & p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi < p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi \\ = 0 & p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi = p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi \\ > 0 & p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi > p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi \end{cases}$.

Or

we have the relationship $\frac{\partial n_f^*}{\partial \omega} \begin{cases} < 0 & \int_0^{\bar{b}} (p\xi_0 \xi - p_f \xi^2) u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi < 0 \\ = 0 & \int_0^{\bar{b}} (p\xi_0 \xi - p_f \xi^2) u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi = 0 \\ > 0 & \int_0^{\bar{b}} (p\xi_0 \xi - p_f \xi^2) u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi > 0. \end{cases}$.

Under $p\xi_0 \int_0^{\bar{b}} \xi u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi < p_f \int_0^{\bar{b}} \xi^2 u''(\xi_0 n_f^* + \xi n^*) f(\xi) d\xi$, we have $\frac{\partial n_f^*}{\partial \omega} < 0$.

Or the corresponding goods are inferior.

Results of Proposition 1 are therefore achieved and the proof is complete. ■

Proof of Proposition 2. Eq. (7) directly implies $\frac{\partial n^*}{\partial \omega} \begin{cases} > 0 & \int_0^{\bar{b}} (p_f \xi - p\xi_0) u''(\xi_0 n_f + \xi n) f(\xi) d\xi > 0 \\ = 0 & \int_0^{\bar{b}} (p_f \xi - p\xi_0) u''(\xi_0 n_f + \xi n) f(\xi) d\xi = 0 \\ < 0 & \int_0^{\bar{b}} (p_f \xi - p\xi_0) u''(\xi_0 n_f + \xi n) f(\xi) d\xi < 0. \end{cases}$ From Eq. (7) and (A.1), we have $p\frac{\partial n^*}{\partial \omega} + p_f \frac{\partial n_f^*}{\partial \omega} = 1$.

Conclusions are obtained and the proof is complete. ■

Proof of Proposition 3. Here we consider the price effects of the bubble free asset.

$$\begin{aligned}\frac{\partial n_f^*}{\partial p_f} &= -\frac{\det\begin{bmatrix}\frac{\partial g_1}{\partial n} & \frac{\partial g_2}{\partial n}\\ \frac{\partial g_1}{\partial p_f} & \frac{\partial g_2}{\partial p_f}\end{bmatrix}}{\det J} = -\frac{\det\begin{bmatrix}-p & \frac{\partial g_2}{\partial n}\\ -n_f^* & -\int_0^{\bar{b}}\xi u'(\xi_0 n_f+\xi n)f(\xi)d\xi\end{bmatrix}}{\det J}\\ &= \frac{p_f n_f^*\int_0^{\bar{b}}\xi^2 u''(\xi_0 n_f+\xi n)f(\xi)d\xi - p n_f^*\xi_0\int_0^{\bar{b}}\xi u''(\xi_0 n_f+\xi n)f(\xi)d\xi - p\int_0^{\bar{b}}\xi u'(\xi_0 n_f+\xi n)f(\xi)d\xi}{\det J}\\ &= \frac{\int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f+\xi n) - pu'(\xi_0 n_f+\xi n)]f(\xi)\xi d\xi}{\det J}.\end{aligned} \tag{A.2}$$

Therefore,

$$\frac{\partial n_f^*}{\partial p_f}\begin{cases} > 0 & \int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f+\xi n) - pu'(\xi_0 n_f+\xi n)]f(\xi)\xi d\xi > 0\\ = 0 & \int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f+\xi n) - pu'(\xi_0 n_f+\xi n)]f(\xi)\xi d\xi = 0\\ < 0 & \int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f+\xi n) - pu'(\xi_0 n_f+\xi n)]f(\xi)\xi d\xi < 0.\end{cases}$$

Therefore, the bubble free asset is Giffen good under the conditions of $\int_0^{\bar{b}}[n_f^*(p_f\xi - p\xi_0)u''(\xi_0 n_f+\xi n) - pu'(\xi_0 n_f+\xi n)]f(\xi)\xi d\xi > 0$.

Conclusions are achieved and the proof is complete. ■

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